Stoke's Measurement- Sinusoidal Driving Voltage

Daniel Fenstermaker, Nicholas Hall

8/21/2014

Mentioned in the Spring 2014 Stoke's Measurement method, a piezo-controlled translation stage is driven with a function generator to vary the velocity of the trapped particle. The trapping force is then determined using the relationship:

$$F_d = 6\pi\mu R v_s$$

Where R is the radius of the particle, μ is the dynamic viscosity of the fluid (in this case, water at room temperature), and v_s is the velocity of the particle when it escapes the trap. The idea is that when the particle barely breaks free from the trap, Stoke's drag force and the beam trapping force are equal.

Using a **sinusoidal driving voltage** for the piezo-controlled translation stage can be a better method for determining this terminal velocity for several reasons:

- When the maximum velocity for a harmonic oscillator in one dimension is achieved, the instantaneous acceleration is **zero**. This assures that momentum-change has no play in the measurements.
- At the maximum positions of the trajectory (at the driving voltage's maximum and minimum value) the particle is slowed down to zero velocity. This assures that the particle does not break free due to instantaneous acceleration at the foot of a step-function driving voltage or ramp driving voltage.
- The harmonic motion of the particle can be repeatedly observed as the frequency of motion is slowly increased

Driving the piezo-controlled translation stage in one dimension can then be analyzed using the equations of simple harmonic motion. **A relationship between driving frequency and maximum velocity can be determined.** Remember, the amplitude "A" will be one quarter of the total distance traversed by the particle in one full period. If the translation stage is moving forward 20 microns and then back 20 microns, the amplitude of this motion is 10 microns!!!

Simple harmonic motion:

$$\text{Position: } x = A\sin(\omega t)$$
$$\text{Velocity: } v = \omega A\cos(\omega t)$$
$$v_{max} = \omega A = 2\pi f A$$

$$\boldsymbol{F_d = 12\pi^2 \mu R f}$$

When the terminal frequency is reached, it will appear that the particle is jumping in and out of the trap. A video is included on the wiki site.